

# Estimating the Spreading Potential of an Epidemic When Observations Are Scarce

Andrea Kraus & Victor M. Panaretos

*Section de Mathématiques  
Ecole Polytechnique Fédérale de Lausanne*

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## Abstract

We consider the problem of inferring the potential of an epidemic for escalating into a pandemic on the basis of limited observations in its initial stages. Classical results of Becker & Hasofer [(1997), *J. Roy. Statist. Soc. B*] illustrated that, surprisingly, frequentist estimation of the complete set of parameters of an epidemic modelled as a birth and death process remains feasible even when one is only able to observe the deaths and the total number of births. We consider a more realistic scenario, where observations are severely more limited: observing only a random proportion of the deaths at discrete time intervals. Although the individual parameters of the birth and death process are no longer estimable, we nevertheless demonstrate that estimation of the Malthusian parameter governing the growth of the epidemic is still feasible. We construct explicit and straightforwardly calculable estimators motivated heuristically by the martingale dynamics of the process, and show that they admit a rigorous quasi-likelihood interpretation. We establish the consistency and asymptotic normality of these estimators, allowing for the construction of approximate confidence intervals that can be used to infer the spreading potential of the epidemic. A simulation study reveals that the estimation is efficient even under scarce observation. We illustrate our methods on the initial outbreak data of the 2009 H1N1 influenza pandemic.

**Keywords:** *Birth and death process; Malthusian parameter; Markov process; marked point process; martingale; explosion; extinction; estimating equation; quasi-likelihood.*