A Delta Method for Empirical Evolution Equations

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Abstract

An evolution equation on a Banach space X models the time evolution of a system with state-space X, implicitly defining a flow in that space by relating the time derivative of a parametrised curve in X to a vector field on X. Such equations encompass a very rich variety of models in the physical sciences: when X is finite-dimensional, evolution equations encode systems of ordinary differential equations; and, in the infinite dimensional case, evolution equations are partial differential equations. In practice, evolution equations may depend on quantities that are unknown and need to be estimated. We consider the statistical setting where the vector field defining the equation is unknown and has to be nonparametrically estimated. Calling such equations *empirical evolution equations*, we set out to describe how the uncertainty in the equation itself propagates to its solution. We prove results for general Banach spaces that allow one to push forward functional limit theorems on the vector field defining the equation, to functional limit theorems on the solution of the equation itself, obtaining what can effectively be considered a delta method. Furthermore, we illustrate the range of applicability of these results, by means of two concrete case studies: one in astrophysics, and one in brain imaging.

Keywords: *Functional limit theorem; gradient flow; integral curve; random field; uncertainty quantification.*