Statistical Inference on the Second-Order Structure of Functional Data in the Presence of Influential Observations

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Abstract

Inferences related to the smoothness properties of functional data, as expressed by second-order structure, can become unreliable when the data contain unusual observations. In the functional setting, it is often difficult to identify atypical observations as their distinguishing characteristics can be manifold but subtle. Visually, they often appear to not be globally far from the main part of the data. Yet such observations may have serious influence on the stability of statistical methods due to internal deviations from typical observations. In this paper we develop a test for comparing the second-order characteristics of two samples of functional data which is resistant to atypical observations. The proposed test is a regularised M-test based on a spectrally truncated version of the Hilbert–Schmidt norm of a score operator. We derive the asymptotic distribution of the test statistic, investigate the behaviour of the test in a simulation study and illustrate the method on a real dataset of DNA minicircle strands, observed through the electron microscope.

Keywords: *Covariance Operator; Dispersion Operator; Functional Data Analysis; Karhunen-Loève Expansion; M-Estimation; Resistant Test; Spectral Truncation; Two-sample testing.*