Fourier Analysis of Stationary Time Series in Function Space

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Abstract

We develop the basic building blocks of a frequency domain framework for drawing statistical inferences on the second-order structure of a stationary sequence of functional data. The key element in such a context is the spectral density operator, which generalises the notion of a spectral density matrix to the functional setting, and characterises the second-order dynamics of the process. Our main tool is the functional Discrete Fourier Transform (fDFT). We derive an asymptotic Gaussian representation of the fDFT, thus allowing the transformation of the original collection of dependent random functions into a collection of approximately independent complex-valued Gaussian random functions. Our results are then employed in order to construct estimators of the spectral density operator based on smoothed versions of the periodogram kernel, the functional generalisation of the periodogram matrix. The consistency and asymptotic law of these estimators are studied in detail. As immediate consequences, we obtain central limit theorems for the mean and the long-run covariance operator of a stationary functional time series. Our results do not depend on any structural modeling assumptions, but only on functional versions of classical mixing conditions. We give examples of functional processes satisfying our mixing conditions, conduct a simulation study to assess finite sample performance and study the effect of observing the functions on finite grids.

Keywords: *cumulants; discrete Fourier transform; functional data analysis; functional time series; mixing; periodogram kernel; spectral density operator.*