Smooth Plus Rough Variation of Random Functions: The Interplay Between Rank, Resolution, and Scale

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Technical Report # 01/15, December 2015
Chair of Mathematical Statistics

Abstract

Functional data analyses typically proceed by smoothing, followed by functional PCA. This paradigm implicitly assumes that any roughness is due to nuisance noise. Nevertheless, relevant functional features such as time-localised or short scale variations may indeed be rough. These will be confounded with the smooth components of variation by the smoothing/PCA steps, potentially distorting the parsimony and interpretability of the analysis. The goal of this paper is to investigate how both smooth and rough variations can be recovered on the basis of discretely observed functional data. Assuming that a functional datum arises as the sum of two uncorrelated components, one smooth and one rough, we develop identifiability conditions for the estimation of the two corresponding covariance operators. The key insight is that they should possess complementary forms of parsimony: one smooth and low rank (large scale), and the other banded and potentially high rank (small scale). Our conditions elucidate the precise interplay between rank, bandwidth, and grid resolution. We construct nonlinear estimators of the smooth and rough covariance operators and their spectra via matrix completion, without assuming knowledge of the true bandwidth or rank; we establish their consistency and rates of convergence, and use them to recover the smooth and rough components of each functional datum, effectively producing separate functional PCAs for smooth and rough variation.

Keywords: *Covariance operator; functional data analysis; functional PCA; matrix completion; low rank; banding; smoothing*