Hybrid Regularisation of Functional Linear Models

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Abstract

We consider the problem of estimating the slope function in a functional linear model with a scalar response and a functional covariate. This central problem of functional data analysis is well known to be ill-posed, thus requiring a regularised estimation procedure. The two most commonly used approaches, based on spectral truncation or Tikhonov regularisation of the empirical covariance operator, have both been shown to have strengths and weaknesses depending on the precise setting. From a mathematical viewpoint, Tikhonov regularisation appears to be the more canonical choice, as it is robust to eigenvalue ties, while it also attains the asymptotically optimal minimax rate of convergence in mildly ill-posed settings. In this paper, we show that, surprisingly, one can strictly improve upon the performance of the Tikhonov estimator while retaining its stability properties by combining it with a form of spectral truncation. We prove that this hybrid estimator enjoys the same minimax optimal rates as the Tikhonov estimator, but that it strictly and uniformly improves upon it in a finite sample sense, for all sufficiently large sample sizes. The hybrid estimator is straightforward to construct in practice, and we investigate its performance by means of a simulation, demonstrating that one can make substantial gains even in finite samples.

Keywords: *Functional data analysis; ill-posed problem; mean squared error; principal component analysis; rate of convergence; ridge regression; spectral truncation; Tikhonov regularisation.*